



# Calculus Cheat Sheet

## Derivatives

### Definition and Notation

If \_\_\_\_\_ then the derivative is defined to be  $\lim_{h \rightarrow 0} \frac{\text{_____}}{\text{_____}}$ .

If \_\_\_\_\_ then all of the following are equivalent notations for the derivative.

— — —

If \_\_\_\_\_ all of the following are equivalent notations for derivative evaluated at \_\_\_\_\_.

| —| —|

### Interpretation of the Derivative

If \_\_\_\_\_ then,

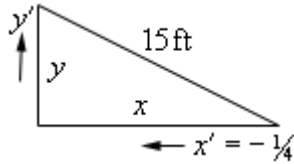




**Related Rates**

Sketch picture and identify known/unknown quantities. Write down equation relating quantities and differentiate with respect to using implicit differentiation ( add on a derivative every time you differentiate a function of ). Plug in known quantities and solve for the unknown quantity.

**Ex.** A 15 foot ladder is resting against a wall. The bottom is initially 10 ft away and is being pushed towards the wall at  $\frac{1}{4}$  ft/sec. How fast is the top moving after 12 sec?



$x'$  is negative because  $x$  is decreasing. Using Pythagorean Theorem and differentiating,

$$y^2 + x^2 = 15^2 \quad 2y y' + 2x x' = 0$$

After 12 sec we have  $x = 10 - 12 \cdot \frac{1}{4} = 7$  and

so  $y = \sqrt{15^2 - 7^2} = \sqrt{176}$ . Plug in and solve for  $y'$ .

$$7 \cdot \frac{1}{4} + \sqrt{176} \cdot y' = 0 \quad y' = -\frac{7}{4\sqrt{176}} \text{ ft/sec}$$

**Ex.** Two people are 50 ft apart when one starts walking north. The angle  $\theta$  changes at  $0.048 \text{ rad/sec}$ . At what rate is the distance between them changing when  $\theta = 0.6$ ?

## Integrals Definitions

**Definite Integral:** Suppose  $f$  is continuous on  $[a, b]$ . Divide  $[a, b]$  into  $n$  subintervals of width  $\Delta x$  and choose  $x_i^*$  from each interval. **Anti-Derivative :** An anti-derivative of  $f$  is a function,  $F$ , such that  $F'(x) = f(x)$ .

Then  $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x$ .

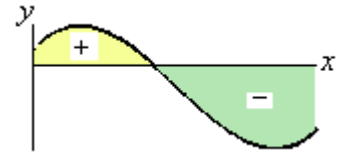




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### Applications of Integrals

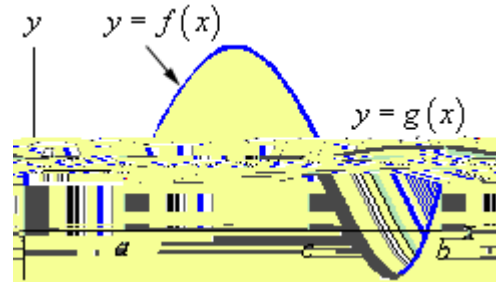
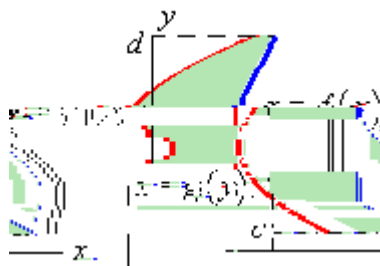
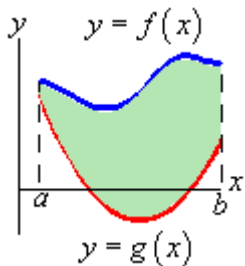
**Net Area :** represents the net area between and the -axis with area above -axis positive and area below -axis negative.



**Area Between Curves :** The general formulas for the two main cases for each are,

$$\int_a^b (\text{upper function}(x) - \text{lower function}(x)) dx \quad \& \quad \int_c^d (\text{right function}(y) - \text{left function}(y)) dy$$

If the curves intersect then the area of each portion must be found individually. Here are some sketches of a couple possible situations and formulas for a couple of possible cases.



**Volumes of Revolution :** The two main formulas are and . Here is some general information about each method of computing and some examples.

#### Rings

$$2\pi \int_a^b (\text{outer radius}^2 - \text{inner radius}^2) dx$$

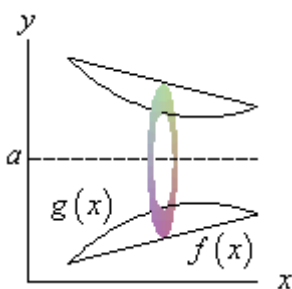
Limits: / of right/bot ring to / of left/top ring  
 Horz. Axis use , Vert. Axis use ,  
 , and . , and .

#### Cylinders

$$2\pi \int_a^b \text{radius} \cdot \text{width/height} dy$$

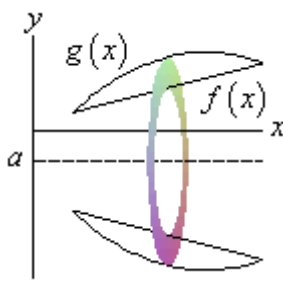
Limits : / of inner cyl. to / of outer cyl.  
 Horz. Axis use , Vert. Axis use ,  
 , and . , and .

**Ex. Axis :** 0



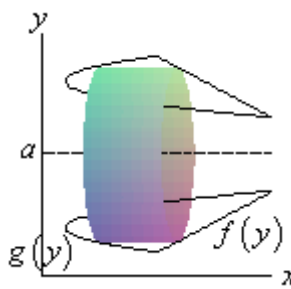
outer radius :  
 inner radius :

**Ex. Axis :** 0



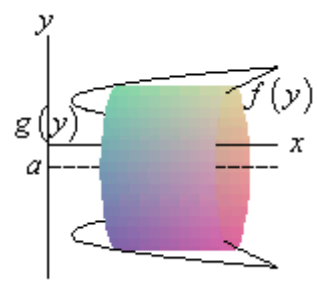
outer radius: ||  
 inner radius: ||

**Ex. Axis :** 0



radius :  
 width :

**Ex. Axis :** 0



radius : ||  
 width :

These are only a few cases for horizontal axis of rotation. If axis of rotation is the -axis use the 0 case with 0. For vertical axis of rotation ( 0 and 0) interchange and to get appropriate formulas.

